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Nicholas G. Horton
Department of Physics Rollins College, nhorton@rollins.edu

Thomas R. Moore
Department of Physics, Rollins College, TMOORE@rollins.edu

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Nicholas G. Horton and Thomas R. Moore
Department of Physics, Rollins College, Winter Park, Florida 32789

The magnetic pickup of an electric guitar uses electromagnetic induction to convert the motion of a ferromagnetic guitar string to an electrical signal. Although the magnetic pickup is often cited as an everyday application of Faraday's law, few sources mention the distortion that the pickup generates when converting the motion of a string to an electric signal, and fewer analyze and explain this distortion. We model the magnet and ferromagnetic wire as surfaces with magnetic charge and construct an intuitive model that accurately predicts the output of a magnetic guitar pickup. This model can be understood by undergraduate students and provides an excellent learning tool due to its straightforward mathematics and intuitive algorithm. Experiments show that it predicts the change in a magnetic field due to the presence of a ferromagnetic wire with a high degree of accuracy. © *2009 American Association of Physics Teachers.*
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I. INTRODUCTION

The electric guitar is one of the most ubiquitous instruments in contemporary music. These instruments utilize a pickup mechanism to translate string vibrations into an electric signal that can be sent to an amplifier or a recording device. Of the many types of guitar pickups available, the magnetic pickup is the most common.

The magnetic pickup is a straightforward application of Faraday's law, and many physics textbooks use it as an example of the practical application of electromagnetic induction.[1–4] However, these books merely mention this application and do not discuss some of the important aspects. Therefore, there is a significant amount of physics and mathematics that can be addressed within the context of this example that is seldom introduced to students. In particular, textbooks neglect the fact that the inherent properties of the pickup distort the signal. This distortion is caused by obvious factors such as the geometry and location of the pickup, and more subtle factors such as the nonuniformity of the magnetic field. Even texts on musical acoustics, which correctly address the importance of the placement of the pickup with respect to the string, ignore the nonlinearity inherent in the process of converting the motion of the string to an electrical signal.[5,6]

A simple magnetic pickup (see Fig. 1) is comprised of a permanent magnet surrounded by a coil of wire with typically several thousand turns. The guitar strings consist of wires made of a ferromagnetic material and are parallel to the face of the magnet. The magnetic field of the pickup/wire system, and thus the magnetic flux through the coil, depends critically on the position of the wire. Therefore, moving the wire changes the magnetic flux through the coil. According to Faraday's law the current induced inside the coil is proportional to the time rate of change of the magnetic flux through the coil. As the string moves through the magnetic field a time-varying current is produced in the coil. This current is used to produce a potential drop across a resistor, which is then amplified and sent to a speaker.

We present here a simple model of an electric guitar pickup. We use this model to study the conversion of string

motion into a change in the magnetic field and show that vertical oscillations of the guitar strings produce a much stronger effect than horizontal oscillations of the strings. Additionally, there is a noticeable distortion of the signals generated by both oscillations.

A model of the electric guitar pickup has recently been reported in the literature.[7] However, this and other models are intended for research purposes and are not suitable for undergraduate students or those who do not have access to advanced magnetic field simulations. Our approach is more intuitive, which helps give insight into the physics of these devices and explains much about why electric instruments sound as they do.

II. THEORY

There are many ways to model the magnetic field created by a distribution of permanent magnets, and there is a large body of literature on the subject. Because guitar pickups are comprised of cylindrical magnets, it is reasonable to treat the
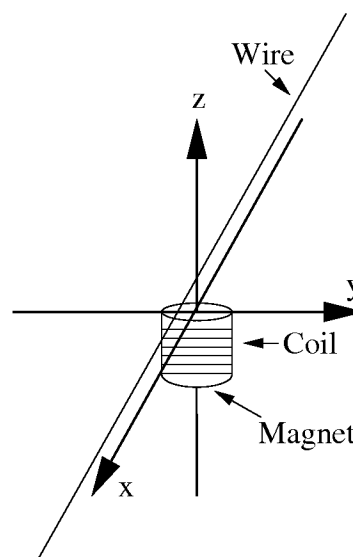

Fig. 1. A simple magnetic pickup. The oscillating ferromagnetic wire induces a change in the magnetic flux through the coil.
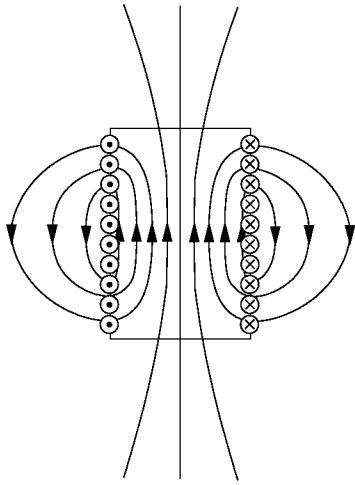
Fig. 2. The field of a permanent magnet approximated as the field produced by a solenoid.

Fig. 3. Field of a permanent magnet approximated as the field produced by magnetically charged plates.

magnets in the pickup as solenoids. The induction field created using this model is displayed in Fig. 2. This model is also easily verifiable because the magnetic field of a permanent magnet can be compared experimentally to the magnetic field of a solenoid. However, the theoretical treatment of the magnetic field due to a solenoid of finite length is very complicated.

Another approach is to use an intuitive, albeit nonphysical, model to calculate the induced magnetic field created by magnets. In this model the north and south ends of a permanent magnet are approximated as magnetically charged plates, with the opposite sides of each magnet having a charge equal in magnitude and opposite in sign.[8,9] Although this model is not physical because the divergence of the magnetic induction field at each disk is nonzero, we will show that it can correctly model the magnetic field outside of the magnet. This model leads to magnetostatic equations similar in form to those of electrostatics, making it easy for students to understand and easy to solve numerically. The field created in this model is shown in Fig. 3.

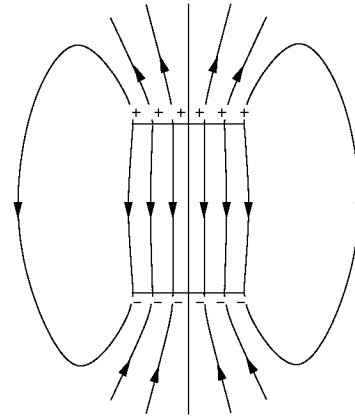The magnetic induction field due to a magnetic point charge is given by

$$\mathbf{B} = B_0 \frac{1}{r^2} \hat{\mathbf{r}}, \tag{1}$$

where r is the distance between the charge and an arbitrary point and B_0 is the field strength of the magnetic monopole. Because a magnetic guitar pickup only detects changes in the magnetic flux through the coil, we will only concern ourselves with the component of the field perpendicular to the face of the magnet and normal to the area enclosed by the coil, which we take to be the z-direction. For these conditions the vertical component of the field at a distance r from the magnetic source is

$$B_z = B_0 \frac{\Delta z}{r^3}, \tag{2}$$

where Δz is the vertical displacement of the observation point from the source.

Figure 4 depicts one face of a cylindrical magnet of radius ψ centered at the point (x,y,z); the face of the magnet is perpendicular to the z-axis. If we assume that the magnetic charge density σ is uniform, the field projected onto the z-axis at (x',y',z') is

$$B_z(x',y',z') = \int_0^{2\pi} \int_0^{\psi} \frac{\sigma(z'-z)\rho}{[(x'-[x-\rho\cos(\phi)])^2 + (y'-[y-\rho\sin(\phi)])^2 + (z'-z)^2]^{3/2}} \, d\rho \, d\phi. \tag{3}$$

The z-component of the magnetic field in free space due to a magnet can be calculated from Eq. (3) by approximating the magnet as two disks of equal and opposite charge separated by the length of the magnet. If there is more than one magnet in the system under consideration, Eq. (3) is solved for each magnetic disk and the total field at an arbitrary point is calculated as the linear superposition of the fields. We know of no closed form solution to Eq. (3), but it can be solved nu-

merically using software readily available to most undergraduates. The examples to follow were solved using MathCad.

Although this model can be used to determine the magnetic field created by any simple system of permanent magnets, the presence of a ferromagnetic wire significantly alters the magnetic field. The situation shown in Fig. 5 describes the model used to determine the magnetic field at (x_p, y_p, z_p)

induced by one disk of uniform magnetic charge density and a wire made of a ferromagnetic material. We model the wire as a series of infinitesimally wide magnets whose strength is linearly proportional to the local field at the position of the wire due to the permanent magnet, and the height of these infinitesimal magnets is equal to the diameter of the wire.

We assume that the coercive field of the permanent magnet is sufficiently large so that the presence of the wire does not affect the magnetic charge density on the surface. Therefore, the only change in the magnetic field at (x_p, y_p, z_p) due

to the presence of the wire is caused by the magnetization of the wire itself. We further assume that the magnetic intensity at the wire is less than is required for saturation, and we ignore the effects of hysteresis. Therefore, the magnetic field of the permanent magnet results in a linear change in magnetization at the wire.

With these assumptions the magnitude of the magnetic induction field at the position (x', y', z') of an infinitesimally wide disk of ferromagnetic material is

$$|\mathbf{B}_w| = \int_0^{2\pi} \int_0^{\psi} \frac{\sigma\rho}{[(x' - [x - \rho\cos(\phi)])^2 + (y' - [y - \rho\sin(\phi)])^2 + (z' - z)^2]} \, d\rho \, d\phi. \tag{4}$$

Because the magnetic field induced by the presence of the ferromagnetic material is proportional to the magnetic field at the wire due to the permanent magnet, the z-component of the magnetic induction field at (x_p, y_p, z_p) due to the infinitesimal section of the wire at (x', y', z') is

$$B_{w,z} = \gamma|\mathbf{B}_w| \frac{(z' - z_p)}{[(x' - x_p)^2 + (y' - y_p)^2 + (z' - z_p)^2]^{3/2}}, \tag{5}$$

where γ is the constant of proportionality relating the local magnetic field produced by a section of the wire to the field induced at (x', y', z') by the permanent magnet (that is, γ is proportional to the magnetic susceptibility of the wire). To determine the field at (x_p, y_p, z_p) due to a length of ferromagnetic wire, the wire must be divided into a finite set of points and $B_{w,z}$ must be calculated for each point. Because guitar strings are ferromagnetic, the presence of a strong positive magnetic charge below the wire induces a negative charge along the bottom of the wire and a positive charge along the top.

In the following we validate this model by comparing the predictions to experimental measurements and then use it to investigate the properties of the electric guitar pickup.

III. EXPERIMENTS AND MODELING

To validate the model we first simulate the magnetic field of a single permanent magnet and compare the predictions of the model with measurements of the field. We then introduce a ferromagnetic wire into the system and compare the measured change in the magnetic field to the predictions of the model. After validating the model, we will use it to study the magnetic field induced by an oscillating wire of ferromagnetic material and calculate the resulting output from the coil of a pickup.

Measurements of the magnetic field were made with a DC magnetometer (Hall probe) manufactured by AlphaLab. The measurements were made in air so that it may reasonably be assumed that the magnetic field is proportional to the magnetic induction field. The magnetometer had a digital readout with a resolution of 0.1 G; the area of the probe was 0.11 ± 0.01 cm^2. For most of the experiments described here, the magnetic field was measured using the display on the magnetometer. The Hall probe was small enough that modeling its area as infinitesimal was appropriate. The probe was oriented so that the magnetic flux through its area was proportional to the z-component of the magnetic field defined in Figs. 4 and 5. The experiments were performed on a wooden table, and the uncertainties in the measurements were smaller than the size of the symbol used in the graphs.
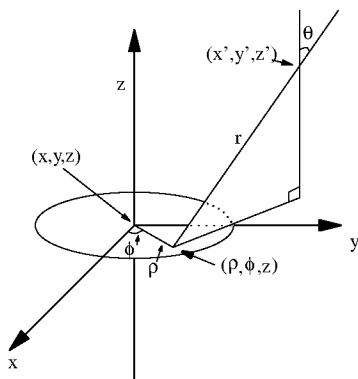

Fig. 4. The coordinate system used for one disk. A magnet is modeled as a set of two magnetically charged disks so that the total magnetic charge for each magnet is zero. The magnetic field is calculated at (x', y', z').
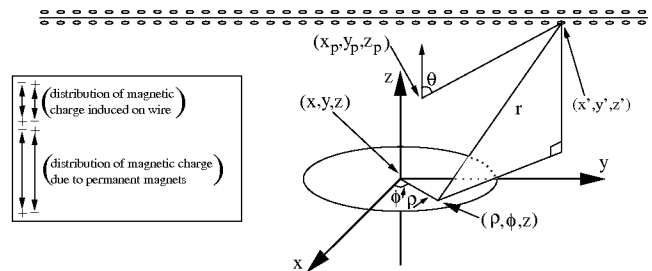

Fig. 5. The coordinate system used for a wire in the presence of a single magnetic disk. The wire is treated as a series of magnetic disks of infinitesimal width.
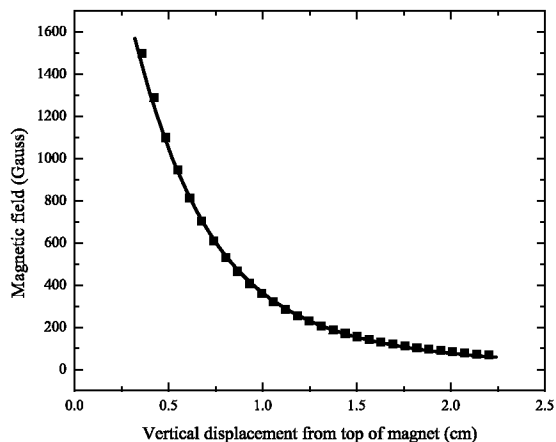
Fig. 6. The measured magnetic field due to a single magnet as a function of the height above the center of the magnet. The line represents the predictions of the model.

A. Single magnet

The simplest system to investigate is a single magnet. The magnet used for this experiment had a diameter of 1.3 ± 0.1 cm and a height of 0.5 ± 0.1 cm. The Hall probe was mounted on an aluminum three-axis translation-stage. Measurements of the magnetic field were made as the probe was moved both vertically and horizontally above the face of the magnet. The measurements and the predictions of the model are shown in Figs. 6 and 7. The magnitude of the maximum field strength was the only fitting parameter. The agreement between the model and experimental results is excellent, especially considering the simplicity of the model.

B. Single magnet and wire

After confirming the validity of the model we investigated the effect of placing a wire in the field of the magnet. Here we are only concerned with the relative change in magnetic flux through an area near the magnet due to the presence of a wire, so we measured the relative change in the magnetic field at the center of the top face of the magnet. The flux through the center of the coil in a guitar pickup is proportional to the field at this point.
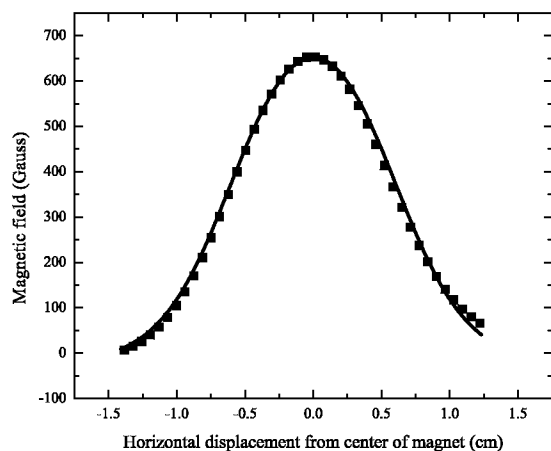

Fig. 7. The measured magnetic field due to a single magnet as a function of horizontal position. The measurements were made 7.1 ± 0.2 mm above the top disk of the magnet. The line represents the predictions of the model.
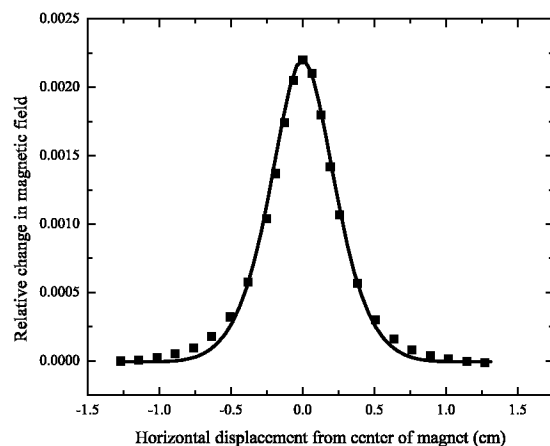

Fig. 8. Relative change in the magnetic field due to a horizontal movement of the wire above a single magnet. The magnetic field was measured on the top surface of the magnet; the wire was 0.4 ± 0.1 mm above the magnet. The line represents the predictions of the model.

The Hall probe was affixed to the top surface of the magnet, and a ferromagnetic wire approximately 23 cm long was used to simulate the presence of the guitar string. The diameter of the wire was 0.8 ± 0.1 mm. The permanent magnet had a diameter of 1.3 ± 0.1 cm and a height of 0.5 ± 0.1 cm. The wire was attached to the translation-stage via an aluminum rod. The magnetic field at the surface of the magnet was measured while the ferromagnetic wire was displaced in both the horizontal and vertical direction.

Although the experiments in Sec. III A can be performed in a rudimentary laboratory, the change in the magnetic field due to a wire requires significantly more sensitivity. To reduce the effects of noise we connected the magnetometer to a computer and at each position of the wire averaged 15 000 measurements of the magnetic field over a 1.5 s interval. Using this method the standard error of the measurements was smaller than the symbols in the graphs.

For this experiment we first measured the magnetic field without the wire on the translation-stage to ensure the stage did not affect the measurement. Our experiment showed that the z-component of the magnetic field did not vary measurably when the translation-stage was positioned at various points with respect to the Hall probe. Subsequently, the wire was added to the aluminum translation-stage and the z-component of the magnetic field was recorded with the wire positioned at various locations above the magnet.

A drift in the output of the magnetometer was observed throughout the duration of the experiment; this drift was linear over the duration of the experiment and the background was adjusted accordingly. Figure 8 shows the magnetic field at the surface of the magnet as a function of the horizontal position of the wire, along with the predictions of the model. Figure 9 shows the magnetic field as a function of the vertical position of the wire. In both cases the only fitting parameter was the maximum value of the field. The predictions of the model are in agreement with the measurements.

C. Modeling multiple magnets

We measured the magnetic field of a Basslines SPB-3 Quarter-Pound magnetic bass pickup to determine if our model accurately predicts the field of a pickup that contains multiple magnets. A diagram of the pickup is displayed in
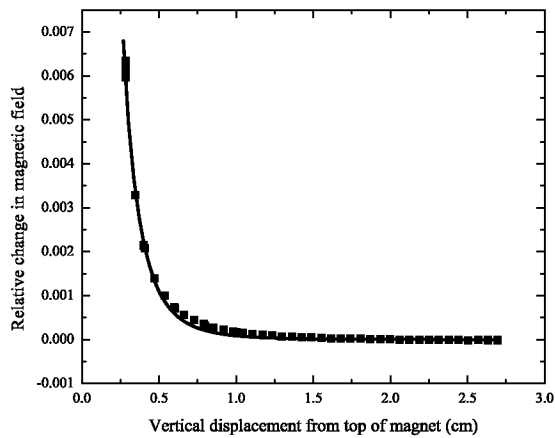
Fig. 9. Relative change in the magnetic field due to a vertical movement of the wire above a single magnet. The magnetic field was measured on the top surface of the magnet. The line represents the predictions of the model.

Fig. 10. The height of each magnet was 1.8 ± 0.1 cm; the spacing between the centers of the magnets was 1.0 ± 0.1 cm. The orientation of the magnetic field produced by each of the four cylindrical magnets was the same. The relative strength of each magnet was measured and factored into the model by adjusting the value of σ for each magnet. The coil was removed from around the pickup and the origin was defined as shown in Fig. 10. The origin was located 1 mm below the top surfaces of the magnets and coincident with the upper casing of the pickup. We measured the z-component of the magnetic field as a function of displacement from the origin in the x-, y-, and z-directions. Results of the measurements along the z-axis are shown in Fig. 11(a). The measured results as a function of displacement in the x-direction and y-direction, 0.8 cm above the origin, are shown in Figs. 11(b) and 11(c), respectively. As before, only the maximum value of the magnetic field was used to fit the predictions of the model to the measurements.

The magnetic field measurements and calculations displayed in Figs. 6–9 agree very well with the predicted values, but the measurements and calculations in Fig. 11(a) diverge significantly within 5 mm of the top surface of the magnet. This discrepancy is due to a breakdown of the model when the magnetic field is calculated at a point close to the plane of a disk of magnetic charge but displaced from the surface of the magnet. If the z-component of the magnetic
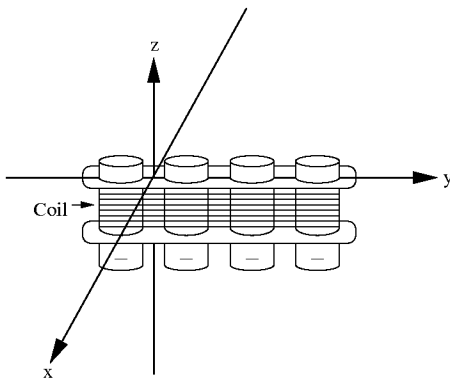

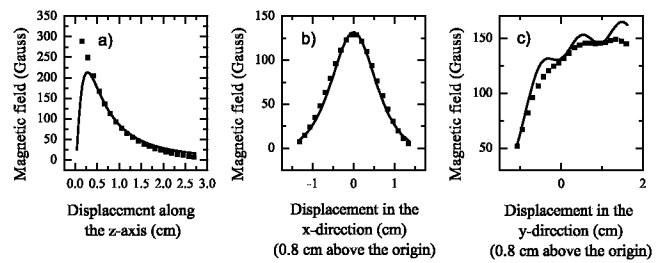Fig. 11. Calculated and measured z-component of the magnetic field due to the bass pickup shown in Fig. 10 as a function of displacement along the (a) z-axis, (b) x-axis, and (c) y-axis.

field is calculated on the same plane as a magnetically charged disk, but laterally displaced, the charge on the disk makes no contribution to the z-component of the magnetic field due to the non zero divergence of the field. At locations significantly above the magnetically charged disks, or those close to the plane of the disk but not displaced laterally, the calculated z-component of the magnetic field is consistent with the measured field. Fortunately, the strings of electric instruments are placed significantly above the top surface of the pickup and in these cases the field can be accurately predicted by the model.

D. Modeling the electric guitar pickup

The model was used to calculate the z-component of the magnetic field at the surface of the magnet in the presence of a sinusoidal oscillation of the wire above it. According to Faraday's law, the time-derivative of this field is proportional to the current induced in a coil surrounding that point. The amplitudes of the oscillations were assumed to be 1 mm. Figure 12 displays the calculated z-component of the magnetic field at the top surface of a single cylindrical magnet due to a wire oscillating 4 mm above it. Figure 13 displays the calculated z-component of the magnetic field 1 mm above the origin of the pickup used in Sec. III C due to a
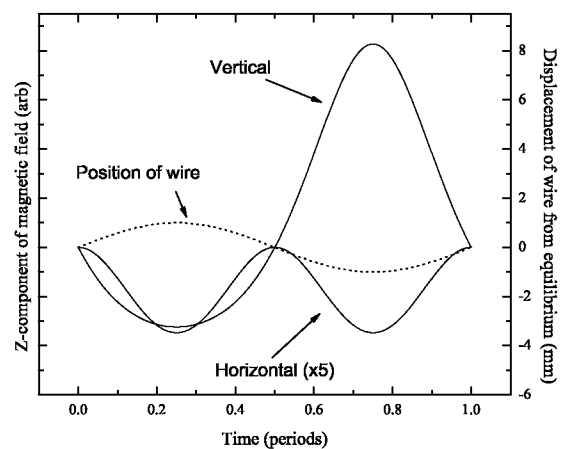

Fig. 12. Calculated z-component of the magnetic field due to a wire oscillating 4 mm above the top face of a single permanent magnet. The amplitude of the vibrating wire is 1 mm and the field is calculated at the center of the top face of the magnet. The curve labeled "horizontal" denotes the field generated by horizontal oscillations of the wire and the curve labeled "vertical" denotes the field generated by vertical oscillations of the wire. The curve representing the results due to horizontal oscillations has been magnified by a factor of 5.
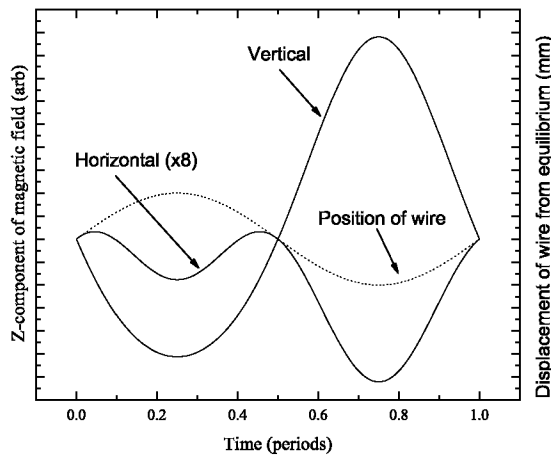

Fig. 10. Drawing of a commercially available multi-magnet magnetic pickup.

Fig. 13. Calculated z-component of the magnetic field due to a wire oscillating 8 mm above origin of the bass pickup shown in Fig. 10. The amplitude of the vibrating wire in the model is 1 mm and the field is calculated at a position above the origin and on the same xy-plane as the top surface of the magnets. The curve labeled "horizontal" denotes the field generated by horizontal oscillations of the wire and the curve labeled "vertical" denotes the field generated by vertical oscillations of the wire. Note that the curve representing the result due to horizontal oscillations has been magnified by a factor of 8.

wire oscillating 8 mm above the origin (that is, 7 mm above the surface of the magnets). In each graph the curves labeled "horizontal" and "vertical" represent the z-components of the magnetic field induced by horizontal and vertical oscillations of the wire.

Inspection of Figs. 12 and 13 reveals that the magnetic field produced by a sinusoidally vibrating wire is inherently nonsinusoidal. This distortion alters the sound produced by the musical instrument and is unavoidable when using magnetic pickups. The plot of the z-component of the magnetic field in Figs. 12 and 13 for the horizontal motion of the string has been greatly magnified because the signal produced by a wire oscillating in the vertical direction is significantly greater than that produced by one oscillating in the horizontal direction. There are two maxima of the magnetic field for each horizontal oscillation of the wire because the symmetric
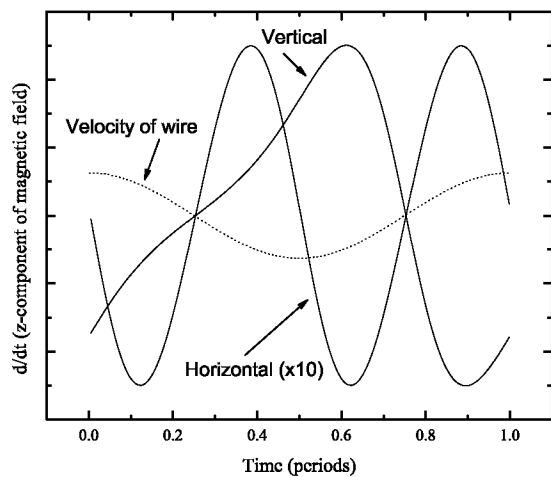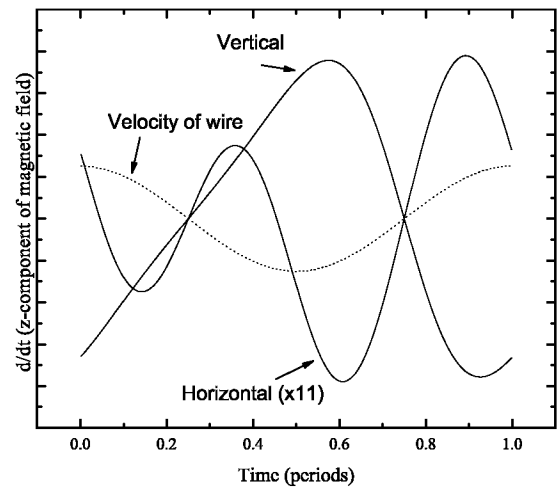

Fig. 15. Time-derivative of curves displayed in Fig. 13. The solid curves refer to the signal generated in the coil when the string is oscillating sinusoidally in the horizontal and vertical directions.

configuration of the magnets in each system acts to double the frequency in the resulting signal. There is also a distinctly nonsinusoidal magnetic field induced by the sinusoidal vertical oscillations of the wire. Each of these distortions results in the presence of higher harmonics in the sound from an electric guitar.

Because magnetic guitar pickups convert the velocity of the guitar string into an electric signal, the time derivatives of the curves in Figs. 12 and 13 determine the current generated in a coil surrounding the magnets. These results are displayed in Figs. 14 and 15, where the curves representing the effects due to the horizontal motion of the wire have been multiplied by a constant. The power spectra shown in Figs. 16 and 17 show that the signal due to the sinusoidal oscillations of the strings of an electric guitar are significantly distorted before being sent to the amplifier.

The predictions of the spectra resulting from a sinusoidally oscillating wire over a pickup using a single-magnet are probably very accurate due to the small area involved, but the signal produced by a large, multi-magnet pickup may be significantly different. To determine the change in flux through a large coil the change in the magnetic field must be

Fig. 14. Time-derivative of curves displayed in Fig. 12. The solid curves refer to the signal generated in the coil when the string is oscillating sinusoidally in the horizontal and vertical directions.
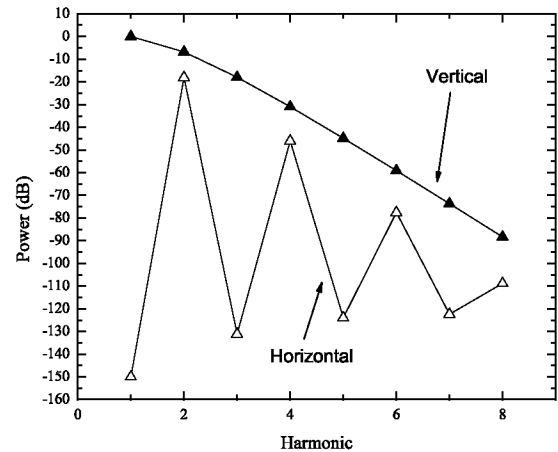

Fig. 16. Power spectra of curves displayed in Fig. 14. The lines are intended only as a guide to the eye.
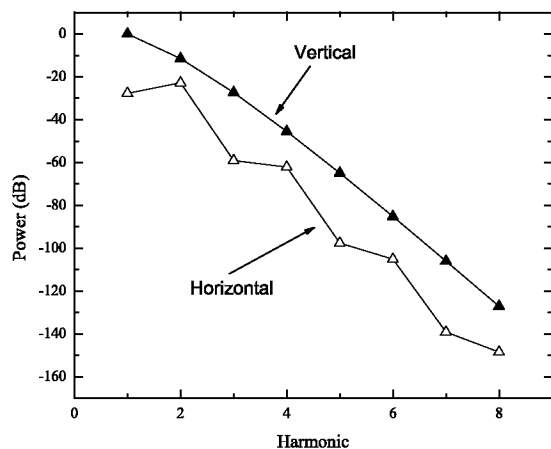
Fig. 17. Power spectra of curves displayed in Fig. 15. The lines are intended only as a guide to the eye.

integrated over the entire area of the coil. Such an integration is not difficult, but is not necessary to understand the origin and form of the distortion created by using magnetic pickups.

IV. DISCUSSION

The magnetic field created by a permanent magnet and a ferromagnetic wire can be modeled by treating the magnet as a pair of disks of opposite magnetic charge and the ferromagnetic wire as a series of infinitesimally wide cylindrical magnets. This simple model can be used to gain insight into how electrical musical instruments work and why their sound is so different from their acoustic counterparts. Although the equations of this model can be derived by undergraduates with only elementary knowledge of electrodynamics, the results of the model agree well with experimental measurements and can be used to investigate why electric instruments sound as they do.

Jungmann has claimed that the electrical signal due to the vertical oscillations of a guitar string are greater in magnitude than that due to horizontal oscillations of the string.[10] The model described here verifies that the signal attributable to the vertical vibrations of an electric guitar string are dominant and also shows that the inherent distortion caused by the nonuniform magnetic field creates significant harmonics in the sound for both vertical and horizontal oscillations of the wire.

Plucked strings always contain harmonics of the fundamental mode of vibration, but the model described here demonstrates that when the wire is part of an electric instrument

each harmonic will also be distorted by the nonlinear properties of the pickup mechanism. The distortion created by the pickup due to the guitar string vibrating at multiple frequencies is beyond the scope of this investigation. However, the effects can be easily predicted by assuming a nonsinusoidal motion of the wire.

As noted, the predictions of the model agree with the experimental results when measurements are made above the center of the magnet. However, the theoretical results deviate from experimental measurements near the surface of and offset from the central axis of the magnet. These deviations are due mainly to the fact that the magnetic induction field of the disks have a nonzero divergence, but also because the body of the magnet is not modeled. Although these simplifications limit the applicability of the model in some situations, it does not significantly affect the study of the electric guitar pickup because the vibrating strings are always above the plane of the magnet. When using this model in an educational setting it is advisable to note this limitation.

We hope that the ease of programming and the intuitive nature of the model will encourage students to design their own pickups and investigate the effects of different geometries. Should students wish to hear the results of their design it is possible to simulate the sound by playing the waveform through a computer soundcard.

ACKNOWLEDGMENT

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